

CFA INSTITUTE RESEARCH FOUNDATION / BRIEF

# INVESTMENT HORIZON, SERIAL CORRELATION, AND BETTER (RETIREMENT) PORTFOLIOS

DAVID BLANCHETT, CFA  
JEREMY STEMPIEN



CFA Institute  
Research  
Foundation

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# CONTENTS

<b>Serial Dependence in Returns</b>	<b>2</b>
<b>Optimal Portfolios over Multiple Periods</b>	<b>6</b>
<b>Conclusion</b>	<b>18</b>
<b>Appendix 1. OLS Regression Results</b>	<b>20</b>
<b>Appendix 2. Expanding the Bootstrap Analysis to International Markets</b>	<b>22</b>
<b>Appendix 3. Pairs Tested for the Respective Factors</b>	<b>23</b>
<b>References</b>	<b>24</b>



# INVESTMENT HORIZON, SERIAL CORRELATION, AND BETTER (RETIREMENT) PORTFOLIOS

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There is notable disagreement among academics about how investment time horizon should potentially affect portfolio allocations. In practice, common portfolio optimization routines—such as mean-variance optimization (MVO), which was introduced by Markowitz (1952)—focus on returns and covariances, where returns are effectively assumed to be independent across time (i.e., follow a random walk). In contrast, many investors and financial advisers believe the risk of owning certain assets, such as equities, declines over longer investment periods, an effect commonly dubbed “time diversification.”

If risk increases proportionally with holding period, an investor’s optimal portfolio will not change (Samuelson 1963); however, if risk does not increase proportionally, the welfare maximizing portfolio would vary. The potential benefit of increasing the allocation to risky assets by time horizon is perhaps best personified by Jeremy Siegel (1994) in the title of his bestselling book *Stocks for the Long Run*.

Using historical US returns, Barberis (2000) and Campbell and Viceira (2003), among others, have demonstrated the long-term value of owning equities, although international evidence is more mixed; Jorion (2003) finds little supporting evidence, and Estrada (2013) notes more positive long-term effects. Recently, using US data back to 1792 and returns created by Dimson, Marsh, and Staunton (2002), McQuarrie (2024) finds mixed evidence that stocks have outperformed bonds, even over relatively prolonged periods.

Research by Bodie (1995) suggests that risks increase over time leveraging concepts around options pricing models, consistent with the cost of insuring against negative returns. Additionally, Pástor and Stambaugh (2012) note that the volatility of stocks increases by time horizon, where the return per annual variance is at least 1.3 times higher than the variance at a one-year horizon, and that estimation errors compound over time.

While these perspectives may appear to be inconsistent, it is important to contrast *absolute* and *relative* risk. On an *absolute* basis, the distribution of compounded wealth is going to increase over time for almost every asset or investment; therefore, it is incorrect to suggest that certain assets (such as stocks) somehow become less risky over longer holding periods (a point that is often incorrectly made referencing the distribution of compounded returns

across investment horizons). However, the rate of change in the distributions of wealth may vary (across and among investments) over time, especially when additional factors, such as inflation, are considered. This means certain assets could become more attractive on a *relative* basis over varying investment periods.

If allocations vary by time horizon, then some type of serial dependence should be present, which means the returns evolve in a way that is not completely random. While research suggests that the return on an investment such as a stock is relatively random, perhaps best exemplified in the title of Burton Malkiel's (2019, 12th ed.) book *A Random Walk Down Wall Street*, it would be incorrect (as we demonstrate) to suggest autocorrelation does not exist at all. Additionally, there could be various relationships across assets, such as inflation, where changes tend to accumulate over longer periods that may not be appropriately captured when focusing on shorter time periods, such as calendar years (or months or quarters).

This paper explores how the allocation to equities, the value and small-cap factors, and commodities varies over different time horizons using historical time-series data. Optimal portfolios are determined using a utility function assuming constant relative risk aversion (CRRA) and are focused on the cumulative growth in wealth over the respective period, which is defined in either nominal or real (i.e., inflation-adjusted) terms.

The analysis demonstrates that optimal portfolios can vary notably over different time horizons, suggesting that serial dependence is an important consideration when building portfolios, especially for more risk averse investors who are focused on growth in real wealth. The effects of inflation are especially striking and suggest that investors more concerned with inflation risk (i.e., retirees) should be especially aware of these effects.

While the analysis suggests that investment professionals need to consider serial dependence when building portfolios, incorporating these relationships into an optimization routine radically increases the complexity, as well as the required assumptions, for effective construction. Additionally, attempting to forecast these underlying relations may seem trivial considering the implied estimation error in forecasting such things as returns (i.e., the first moment of the distribution). Therefore, the implications of serial dependence are likely best considered as part of subjective overlay (i.e., constraint) to existing approaches (e.g., MVO) to account for some of the fundamental factors that are likely to drive returns for different asset classes over time.

## Serial Dependence in Returns

If the expected returns of an asset are defined entirely through the first two moments (return and standard deviation, respectively), the returns would be assumed to be independent over time (and follow a normal or lognormal distribution), consistent with key assumptions used in most portfolio optimization routines today. In reality, the returns of an investment could be related to the past returns of that respective investment (i.e., exhibit autocorrelation) and could co-vary across time with other asset classes.

**Exhibit 1** provides context on the historical autocorrelations for five US asset classes: bills, bonds, stocks, commodities, and inflation, using historical annual returns from 1872 to 2023. The specific sources for each asset class are noted later. Exhibit 1 includes the coefficients from a series of ordinary least-squares (OLS) regressions, where the dependent variable is the actual return for that calendar year while the returns for the previous five calendar years are included as independent variables. Historical returns for each asset class are recentered, so they have an

average return of zero and a standard deviation of 1, to reduce any implications associated with historical differences in returns and risk levels (i.e., the regression is effectively based on the z-values of the historical time-series returns).

Exhibit 1 shows a number of coefficients that are statistically significant, defined as a  $p$ -value of less than 0.05, which suggests the historical returns series is not truly independent across time. Negative coefficients are highlighted in blue, since this implies the risk of the asset effectively declines over time (because a positive return would be more likely to be followed by a negative return), while positive coefficients that are statistically significant are highlighted in red.

For robustness purposes, we repeat this analysis for bill rates, bond returns, and equity returns for each of the 16 countries with available data in the Jordà-Schularick-Taylor (JST) Macrohistory Database. We report the coefficients in Appendix 1.

We can see in Exhibit 1 that certain asset classes, such as bonds, have exhibited positive autocorrelation historically, while other asset classes, such as equities, have exhibited negative autocorrelation. This result suggests that the longer-term risks of owning either asset could change by investment horizon; for example, the relative risk of owning equities should decline compared to bonds.

Next, we demonstrate how the perceived risk of an asset can change by holding period. For this analysis, risk is defined as inflation, where we estimate the correlation between the cumulative growth in wealth and cumulative impact of inflation for different investment horizons for the same four investment asset classes in Exhibit 1 (bills, bonds, equities, and commodities) from 1872 to 2023. The results are included in **Exhibit 2**.

While inflation is often explicitly considered in certain types of optimizations (e.g., a surplus or liability-relative optimization), one potential issue when considering inflation is that changes in the prices of goods or services do not necessarily move in sync with changes in the financial markets (i.e., there could be lagged effects). For example, while financial markets can

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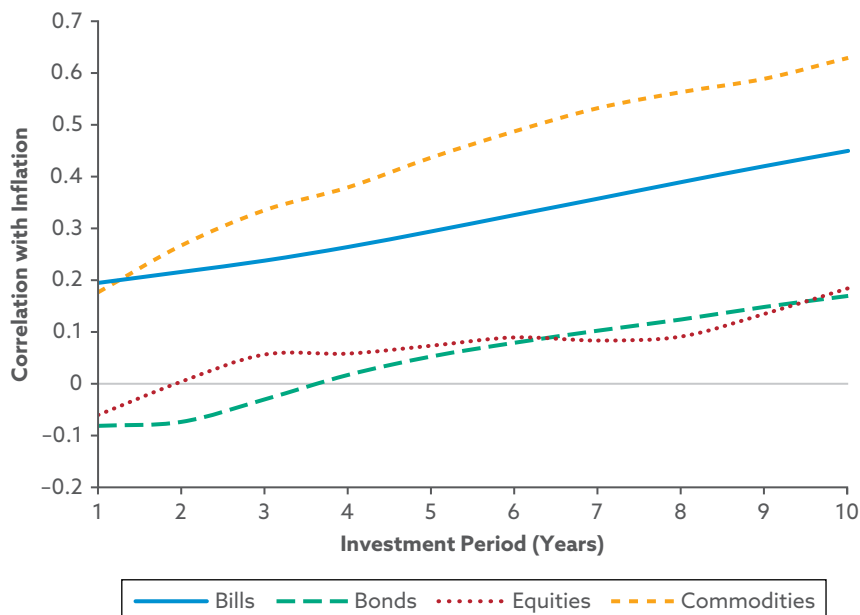
## Exhibit 1. OLS Regression Coefficients Testing for Autocorrelation, 1872–2023

	Bills	Bonds	Stocks	Commodities	Inflation
Intercept	-0.007	-0.011	0.018	0.014	0.021
$t-1$	1.356***	-0.137	0.020	0.196*	0.811***
$t-2$	-0.870***	0.004	-0.225**	-0.081	-0.350**
$t-3$	0.506**	0.200*	0.060	0.020	0.244*
$t-4$	-0.249	0.187*	-0.103	-0.046	-0.183
$t-5$	0.192*	0.037	-0.181*	0.130	0.196*

Note: \*\*\*  $p < 0.001$ , \*\*  $p < 0.01$ , \*  $p < 0.05$ .

Sources: Jordà-Schularick-Taylor (JST) Macrohistory Database (see Jordà, Schularick, and Taylor 2017); Bank of Canada; Morningstar Direct; authors' calculations.

## Exhibit 2. Historical Correlations in Wealth Growth for Various US Asset Classes by Investment Period, 1872–2023



Sources: JST Macrohistry Database; Bank of Canada; Morningstar Direct; authors' calculations.

experience sudden changes in value, inflation tends to take on more of a latent effect, where changes can be delayed and take years to manifest. Focusing on the correlation (or covariance) of inflation with a given asset class (e.g., equities) over one-year periods may hide potential longer-term effects.

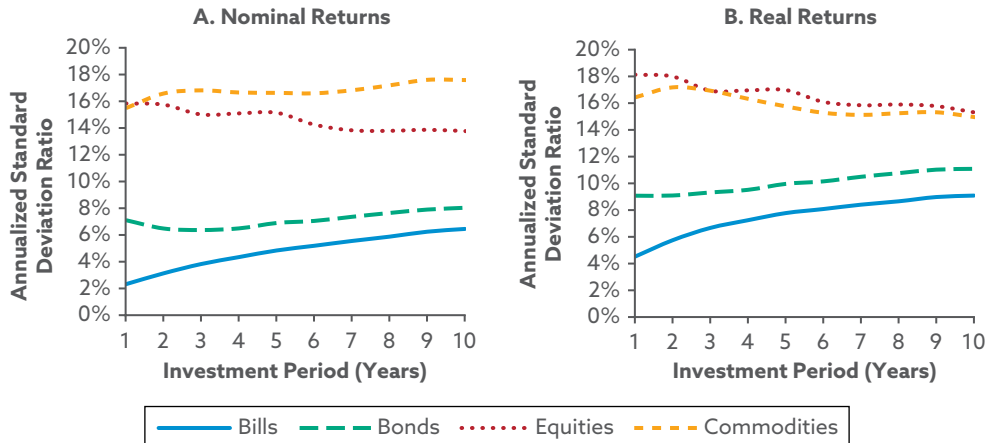
Exhibit 2 demonstrates how the relative correlations of the four asset classes vary notably with inflation by different investment horizons. For example, at a one-year investment horizon, which is a common time frame used for MVO assumptions, the correlations are relatively low for all asset classes, suggesting little potential hedging benefit. However, notable increases occur over a 10-year period (which can be at least partially explained by positive drift for each asset), where the correlation between commodities and inflation increases to 0.62.

The notable increase in correlations for bills and commodities is especially salient because the returns for bills and commodities are notably lower over the historical period (discussed in a future section). This result suggests the effect is not simply due to higher historical returns but, rather, is the result of the differences in how the asset classes have responded to inflation over time.

The results in Exhibits 1 and 2 imply some level of serial dependence among the asset classes considered, which could potentially impact optimal portfolio allocations over longer periods (e.g., 10+ years). This effect is further demonstrated in **Exhibit 3**, which includes information about how the standard deviation of wealth has changed for the respective asset classes across different investment horizons using sequential historical returns from 1872 to 2023.



## Exhibit 3. Standard Deviations of Wealth Changes for Bills, Bonds, Equities, and Commodities for Various Investment Periods, 1872–2023



Sources: JST Macrohistory Database; Bank of Canada; Morningstar Direct; authors' calculations.

The actual historical standard deviations are compared to the deviations from a bootstrap simulation where the historical returns for the respective asset classes are randomly recombined (i.e., bootstrapped). Bootstrapping is useful because it preserves the potentially interesting features of time-series data (e.g., keeps the means and covariances constant) but changes the sequence from the actual historical returns to one that is effectively random. Bootstrapping would capture such things as skewness and kurtosis so that the differences in the wealth distributions would largely be due to some type of serial dependence (e.g., the autocorrelations noted in Exhibit 1).

The first-year (i.e., annual) standard deviation is adjusted based on the ratio of the future standard deviation of terminal wealth values to the bootstrap value for investment periods up to 10 years. If no type of serial dependence in historical returns (e.g., autocorrelation) existed, the lines in Exhibit 3 would be flat; a declining line would suggest negative autocorrelation, and a rising line would suggest positive autocorrelation.

Similar to Exhibits 1 and 2, Exhibit 3 provides evidence that the risk of assets can vary by investment period, especially when considering inflation. For example, in nominal terms (Panel A), the standard deviation of wealth of equities *decreases* over longer investment periods, while the standard deviation of bills, bonds, and commodities *increases*. However, when considering inflation (Panel B), the standard deviation of commodities decreases roughly at the same rate as equities. This is a notable shift and suggests the perceived efficiency of commodities is likely to vary dramatically whether or not inflation is considered (an effect we demonstrate later).

We extend the analysis conducted for Exhibit 3 to international markets and include the results in Appendix 2. Similar to Exhibit 3, the analysis focuses on the ratio of the actual historical standard deviation of wealth to the bootstrapped standard deviation for each of the 14 non-US

countries considered (US data are included for reference purposes). While there is dispersion across countries, the overall results are relatively similar: The actual distribution of wealth tends to decline relative to the bootstrapped values for equities and tends to rise for bonds and bills.

Overall, the analysis so far suggests the optimal allocation to such asset classes as bills, bonds, equities, and commodities could vary over different time periods if the actual historical time series is considered versus assuming the returns are random. This concept is explored in greater detail later.

## Optimal Portfolios over Multiple Periods

Optimal portfolio allocations are determined using a utility function for this research. Adler and Kritzman (2007) suggest utility-based models can be more comprehensive and relevant than defining investor preferences using more common optimization metrics, such as mean and variance. Additionally, Warren (2019) notes that using utility-based methods offers considerable flexibility compared to more traditional approaches since they can be used to “score” different parts of a distribution. More specifically, optimal asset class weights ( $w$ ) are determined that maximize the expected utility ( $U$ ) assuming CRRA, as noted in Equation 1:

$$U(w) = w^{-\gamma} \quad (1)$$

CRRA is a power utility function that is broadly used in academic literature. For example, Thorley (1995) and Warren (2019) both use this approach to demonstrate how the optimal allocation to equities changes by investment horizon.

The analysis assumes varying levels of risk aversion ( $\gamma$ ), where some initial amount of wealth (e.g., \$100) is assumed to grow for some period of time (typically 1 to 10 years, in one-year increments). Higher levels of risk aversion would correspond to investors with lower levels of risk tolerance (i.e., more conservative investors). No additional cash flows are assumed in the analysis.

The analysis uses historical annual, calendar year returns for all calculations, which results in overlapping periods for any kind of multiyear analysis. Note that while monthly returns are available, for some of the data series explored in the analysis (e.g., the small and value factors), we limit the frequency to annual to minimize overlap.

Since risk aversion coefficients can be abstract in nature, risk aversion coefficients are typically converted to a target equity allocation when communicating the results for the analysis. For example, a risk aversion coefficient ( $\gamma$ ) of 2 would generally result in an optimal equity allocation of approximately 50%, depending on the particular test.

**Exhibit 4** provides an example of the utility function in practice. The example includes a scenario with 10 runs that each last five years, where the target wealth value is the terminal value at the end of Year 5. In this example, the returns are randomly generated assuming an average return of 10% and a standard deviation of 20%. The values in Year 5 are each raised to the assumed risk aversion level ( $\gamma$ ), which is 2 in this example. In the first run, the ending balance is \$2.14, and  $\$2.14^{-2} = \$0.22$ . The average utility across the 10 runs is 0.60, and the certainty equivalent wealth value would be \$1.29. If multiple asset classes were being included, the optimal weights among the asset classes considered would be those that maximize the average utility of wealth across the total number of runs.

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### Exhibit 4. Utility Example

A. Returns (%)						B. Growth of \$1						Year 5
Year						Year						$\gamma = 2$
	1	2	3	4	5	0	1	2	3	4	5	
1	17.00	58.63	5.02	-1.81	11.60	\$1.00	\$1.17	\$1.86	\$1.95	\$1.91	\$2.14	0.22
2	5.99	22.14	25.03	-3.58	31.68	\$1.00	\$1.06	\$1.29	\$1.62	\$1.56	\$2.06	0.24
3	32.48	0.04	57.91	14.48	36.38	\$1.00	\$1.32	\$1.33	\$2.09	\$2.40	\$3.27	0.09
4	-9.44	31.79	-11.65	10.63	15.65	\$1.00	\$0.91	\$1.19	\$1.05	\$1.17	\$1.35	0.55
5	35.79	4.18	1.16	-3.39	-6.09	\$1.00	\$1.36	\$1.41	\$1.43	\$1.38	\$1.30	0.59
6	12.11	7.47	24.12	24.80	15.48	\$1.00	\$1.12	\$1.20	\$1.50	\$1.87	\$2.16	0.22
7	29.37	41.04	-25.88	-20.85	-20.23	\$1.00	\$1.29	\$1.82	\$1.35	\$1.07	\$0.85	1.37
8	12.09	-42.89	34.40	-28.46	36.62	\$1.00	\$1.12	\$0.64	\$0.86	\$0.62	\$0.84	1.41
9	-2.47	-4.53	21.45	6.29	-11.27	\$1.00	\$0.98	\$0.93	\$1.13	\$1.20	\$1.07	0.88
10	29.57	0.51	-5.71	26.22	-4.07	\$1.00	\$1.30	\$1.30	\$1.23	\$1.55	\$1.49	0.45
# Run						# Run						
						Average						0.60
						Certainty Equivalent Wealth						\$1.29

Three separate sets of portfolio optimizations were conducted using historical returns, which we detail later. First, optimal allocations to equities more generally are determined using historical returns from 15 different countries. Second, the optimal allocation to the small and value factors within equities is determined. Finally, optimal allocations to commodities are reviewed as proxy for asset classes that could be especially sensitive to whether inflation is considered (i.e., real assets). The respective data sources and particular historical periods for each analysis are included in their respective later sections.

## Allocating to Equities

Optimal equity allocations for various international markets are determined using data from the JST Macrohistory Database.<sup>1</sup> The database includes data on 48 variables, including real and nominal returns for 18 countries from 1870 to 2020. Historical return data for Ireland and Canada are not available, and Germany is excluded given the relatively extreme returns in the 1920s and the gap in returns in the 1940s. This limits the analysis to 15 countries: Australia (AUS), Belgium (BEL), Switzerland (CHE), Denmark (DNK), Spain (ESP), Finland (FIN), France (FRA), the United Kingdom (GBR), Italy (ITA), Japan (JPN), the Netherlands (NLD), Norway (NOR), Portugal (PRT), Sweden (SWE), and the United States (USA).

Four time-series variables are included in the analysis: inflation rates, bill rates, bond returns, and equity returns, where the optimal allocation between bills, bonds, and equities is determined by maximizing certainty-equivalent wealth using Equation 1. Three different risk aversion levels are assumed: low, mid, and high, which correspond to risk aversion levels of 8.0, 2.0, and 0.5, respectively. These, in turn, correspond approximately to equity allocations of 20%, 50%, and 80%, assuming a one-year investment period and ignoring inflation (although the actual resulting allocation varies materially by country). Any year where inflation exceeds 50% (i.e., hyperinflationary periods) is excluded.

**Exhibit 5** includes the optimal equity allocation for each of the 15 countries for five different investment periods: 1, 5, 10, 15, and 20 years, assuming a moderate risk tolerance level ( $\gamma = 2$ ). The optimizations are based on the growth of either nominal wealth or real wealth, using the actual historical sequence of returns or returns that are randomly selected (i.e., bootstrapped) from the historical values, assuming 1,000 trials. The bootstrapping analysis is based on the same returns but effectively assumes they are independent and identically distributed (iid), consistent with common optimization routines, such as MVO.

Exhibit 5 offers a number of important takeaways. First, there are considerable differences in the optimal equity allocations across countries, even when focusing on the return for a single year. For example, the equity allocations range from 16% equities (for Portugal) to 70% (for the United Kingdom). Second, the average equity allocation for the one-year period across all 15 countries is approximately 50% regardless of whether wealth is defined in nominal or real terms. Third, and perhaps most notably, while the equity allocations for the optimizations using actual historical return sequences increase over longer investment optimizations, no change occurs in optimal allocations for the bootstrapped returns. The equity allocations for the nominal wealth optimizations increase to approximately 70% at 20 years, and equity allocations for the real wealth optimizations increase to approximately 80% at 20 years, which

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<sup>1</sup>[www.macrohistory.net/database/](http://www.macrohistory.net/database/).

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### Exhibit 5. Optimal Equity Allocations for a Moderate Risk Aversion Level by Country and Investment Period, 1870–2020

A. Nominal Wealth, Actual Historical Returns						B. Nominal Wealth, Bootstrap Returns							
Country	Investment Period (Years)					Slope	Country	Investment Period (Years)					Slope
	1	5	10	15	20			1	5	10	15	20	
AUS	58.1	75.4	90.8	97.6	100.0	2.2	AUS	49.6	58.0	53.2	61.2	57.5	0.4
BEL	26.8	27.9	38.3	59.4	55.0	1.8	BEL	27.1	23.6	25.8	29.4	25.5	0.1
CHE	48.8	53.8	72.9	77.6	83.6	1.9	CHE	56.0	44.9	49.2	47.4	47.7	-0.3
DNK	59.6	69.8	82.5	83.5	95.7	1.8	DNK	55.3	59.6	60.0	57.9	55.7	0.0
ESP	50.7	44.6	62.6	91.7	95.2	2.9	ESP	55.7	48.8	54.8	51.7	48.9	-0.2
FIN	47.6	53.9	72.2	77.4	87.4	2.1	FIN	54.5	49.3	48.4	51.5	48.2	-0.2
FRA	29.3	23.3	27.8	37.3	37.8	0.7	FRA	25.7	29.6	27.5	29.1	26.7	0.0
GBR	70.2	94.2	100.0	100.0	100.0	1.3	GBR	78.6	72.4	68.7	80.4	76.3	0.1
ITA	32.3	29.0	26.8	28.4	34.8	0.1	ITA	32.3	31.7	31.9	34.0	30.9	0.0
JPN	56.1	38.2	30.7	22.7	17.8	-1.9	JPN	64.5	56.2	57.8	53.8	61.6	-0.2
NLD	52.6	50.0	57.7	53.0	54.1	0.1	NLD	52.7	53.6	49.4	52.4	49.2	-0.2
NOR	42.8	51.3	66.2	67.9	83.5	2.0	NOR	38.9	43.9	48.1	44.1	43.0	0.2
PRT	15.9	15.2	30.4	47.9	54.4	2.3	PRT	7.6	18.4	13.6	15.9	20.2	0.5
SWE	59.4	54.5	57.9	53.7	51.8	-0.3	SWE	67.7	59.6	54.0	56.3	61.6	-0.3
USA	65.0	72.2	87.1	95.7	100.0	1.9	USA	72.4	67.1	67.0	67.7	67.5	-0.2
Avg.	47.7	50.2	60.2	66.2	70.1	1.3	Avg.	49.2	47.8	47.3	48.8	48.0	0.0

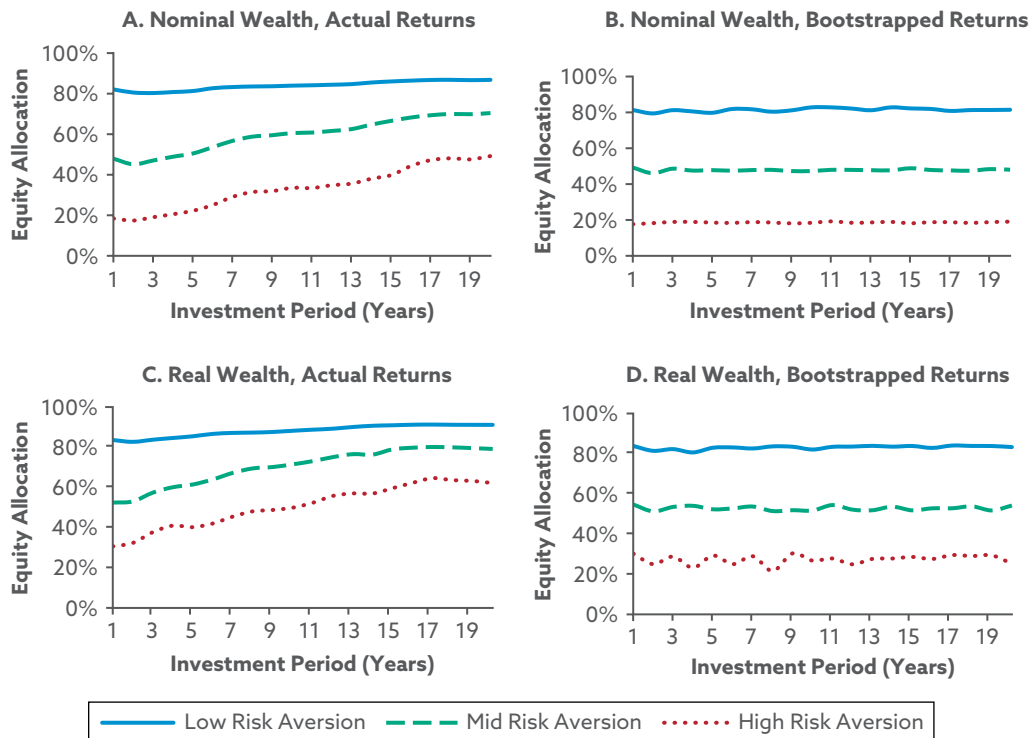
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### Exhibit 5. Optimal Equity Allocations for a Moderate Risk Aversion Level by Country and Investment Period, 1870–2020 (continued)

C. Real Wealth, Actual Historical Returns						D. Real Wealth, Bootstrapped Returns							
Country	Investment Period (Years)					Slope	Country	Investment Period (Years)					Slope
	1	5	10	15	20			1	5	10	15	20	
AUS	55.8	67.2	87.0	97.7	100.0	2.5	AUS	60.0	59.0	56.3	57.1	63.7	0.1
BEL	38.0	65.4	64.9	61.9	58.5	0.7	BEL	37.5	35.4	36.9	37.9	41.9	0.2
CHE	50.0	55.8	74.7	80.4	86.6	2.0	CHE	46.3	48.0	51.5	47.3	51.0	0.2
DNK	62.4	79.6	100.0	100.0	100.0	2.0	DNK	62.1	62.1	60.7	56.5	60.6	-0.2
ESP	53.3	35.5	37.6	62.4	78.4	1.7	ESP	61.2	51.1	51.8	51.3	56.1	-0.2
FIN	62.5	97.3	100.0	100.0	100.0	1.6	FIN	65.5	61.6	42.7	61.9	68.1	0.1
FRA	30.7	8.1	45.8	92.9	75.4	3.7	FRA	31.7	30.2	28.2	29.8	30.2	-0.1
GBR	65.5	77.5	94.3	100.0	100.0	1.9	GBR	60.7	66.8	70.2	57.0	66.0	0.0
ITA	60.9	87.8	87.4	80.7	76.7	0.5	ITA	63.4	56.7	59.1	62.0	65.3	0.2
JPN	57.8	49.1	50.0	48.1	46.7	-0.5	JPN	63.4	61.0	57.5	57.0	61.3	-0.2
NLD	55.3	62.6	73.3	77.7	78.9	1.3	NLD	47.9	55.5	56.0	55.4	53.3	0.2
NOR	47.6	64.9	75.2	79.5	81.6	1.7	NOR	60.9	49.0	53.1	46.2	48.8	-0.5
PRT	17.2	28.9	38.0	46.0	46.6	1.6	PRT	14.0	17.4	16.2	20.3	16.5	0.2
SWE	61.9	62.7	54.8	58.9	57.3	-0.3	SWE	76.8	61.7	65.6	69.8	61.0	-0.5
USA	67.7	79.3	89.3	98.0	100.0	1.7	USA	69.4	69.2	69.9	67.3	69.1	-0.1
Avg.	52.4	61.4	71.5	79.0	79.1	1.5	Avg.	54.7	52.3	51.7	51.8	54.2	0.0

Sources: JST Macrohistory Database; authors' calculations.

## Exhibit 6. Optimal Equity Allocation by Risk Tolerance Level and Investment Period



Sources: JST Macrohistory Database; authors' calculations.

represent annual slopes of 1.3% and 1.5%, respectively. In contrast, the equity allocations for the bootstrapped optimizations are effectively constant (i.e., zero).

This is a key finding: The optimal allocation to equities is different using actual historical return data (which have nonzero autocorrelation) than in the bootstrapped simulation where returns are truly iid.

**Exhibit 6** includes the average allocations to equities across the 15 countries for the three different risk aversion levels when focused on nominal and real wealth and on whether the actual historical sequence of returns are used or if they are bootstrapped. Note that the average values in Exhibit 5 (for the 1-, 5-, 10-, 15-, and 20-year periods) are effectively reflected in the results in Exhibit 6 for the respective test.

Again, we see that optimal equity allocations tend to increase for longer investment periods using actual historical return sequences, but the bootstrapped optimal allocations are effectively constant across investment horizon. The impact of investment horizon using the actual sequence of returns is especially notable for the most risk averse investors. For example, the optimal equity allocation for an investor with a high risk aversion level focused on nominal wealth and a one-year investment horizon would be approximately 20%, which increases to approximately 50% when assuming a 20-year investment horizon.

These results demonstrate that capturing the historical serial dependence exhibited in market returns can affect the optimal allocations. In particular, the optimal allocation to equities tends to increase by investment duration using actual historical returns, suggesting that equities become relatively more attractive than fixed income for investors with longer holding periods.

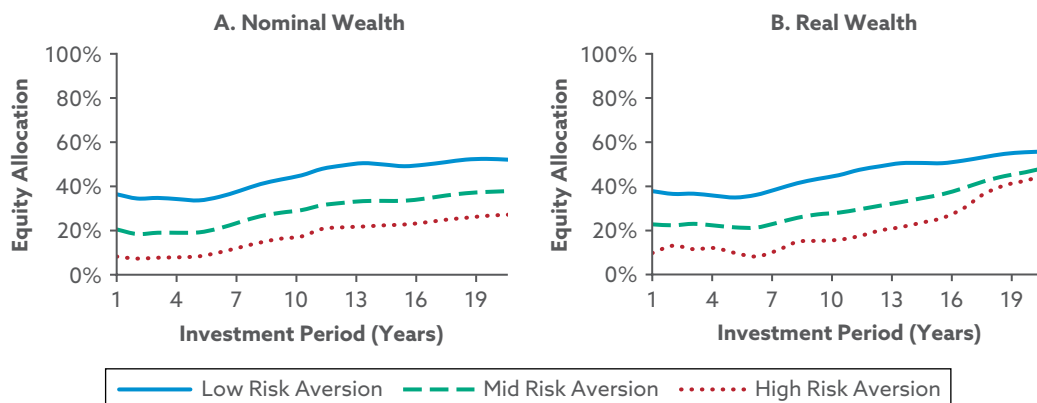
One potential explanation for the change in the optimal equity allocation by time horizon using the actual historical sequence of returns could be the existence of a positive equity risk premium (ERP), which is well documented.<sup>2</sup> To demonstrate how the ERP assumption affects the optimizations, we update the analysis and decrease the return on equities for each country so that the average geometric annual return on equities is the same as the return on bonds for the one-year investment period (for that respective country). In other words, for the purpose of the next exercise, we set the ERP (versus bonds) to zero. The results of these zero-ERP optimizations are reported in **Exhibit 7** for nominal wealth and real wealth in Panels A and B, respectively.

While the equity allocations are notably lower in Exhibit 7 than in Exhibit 6 (focusing on Panel A and Panel C), which we would expect because the assumed return on equities has been significantly reduced, the allocations are not zero, especially as the investment period increases. In other words, even in the absence of an equity risk premium (where equities would be deemed relatively inefficient because they would have the same returns as bonds but with significantly higher risk), equities would still receive an allocation that increases over longer investment periods, especially for more conservative investors focused on inflation risk. This is because even if equities do not provide a return premium, they provide diversification (risk reduction) to a portfolio that does not contain any.

The results in this section suggest that optimization models using one-year returns (e.g., MVO) may overestimate the risk of equities for longer-term investors, especially for more conservative



## Exhibit 7. Optimal Equity Allocation by Risk Tolerance Level and Investment Period When Eliminating the Equity Risk Premium



Sources: JST Macrohistory Database; authors' calculations.

<sup>2</sup>See Siegel and McCaffrey (2023) for an extensive review of the topic.



investors focused on real wealth (i.e., those concerned with inflation risk), to the extent the historical relations persist in the future. In other words, investment horizon and the implications of serial correlation need to be explicitly considered when building portfolios for investors with longer time horizons, especially for more conservative investors who would typically get lower equity allocations.

## Allocating to the Small and Value Factors

Now that we have explored how the optimal allocation to equities varies by investment horizon, we now consider a more granular asset class decision by focusing on two of the most well-known equity factors: size and value. The size effect relates to the fact smaller-capitalization companies have historically outperformed larger-capitalization companies. The companies defined as “value” firms, through such metrics as the book-to-market ratio, typically have outperformed those categorized as “growth” firms, as documented by Fama and French (1993), among others. The size and value factors represent the two dimensions of the relatively popular Morningstar Style Box,<sup>3</sup> which was introduced in 1992.

The outperformance of the small and value factors has declined notably since their initial discovery in the early 1990s, which is unfortunately the case for other factors as well, as documented by McLean and Pontiff (2016), among others. For example, the annualized return of the small factor from 1927 to 1995 was 2.73%, versus 0.46% from 1996 to 2022, and the annualized return of the value factor from 1927 to 1995 was 4.59%, versus –0.11% from 1996 to 2022.

We conducted a series of portfolio optimizations to determine how the optimal allocation to these two factors could change over time. We solve for the allocation between the two factors (i.e., value/growth and large/small) in one-year increments from 1 to 10 years assuming a risk coefficient of 1, which would be consistent for a relatively aggressive investor who would be allocating to these assets (within an equity sleeve).

The analysis uses the “6 Portfolios Formed on Size and Book-to-Market (2 × 3)” dataset available from Kenneth French’s Data Library.<sup>4</sup> The dataset includes historical returns for six asset classes: small growth, small blend, small value, large growth, large blend, and large value. The analysis uses the value-weighted annual returns and involves five different potential pairs of investment options, which are included in Appendix 3.

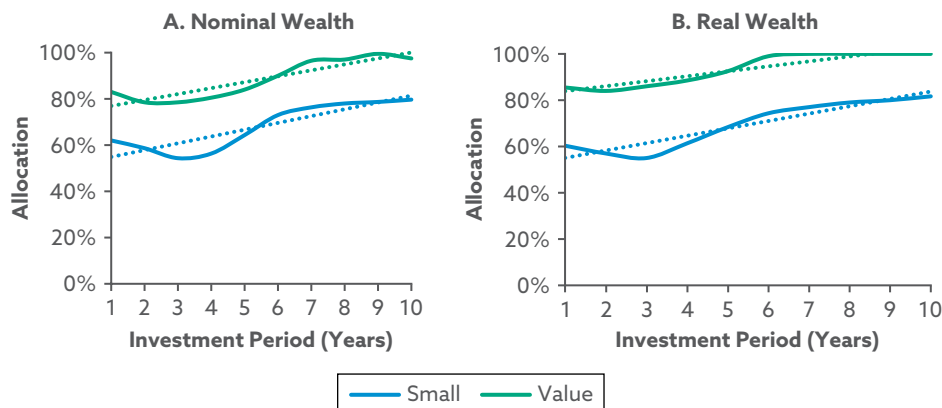
For each pair, the optimal allocation between the two options is determined. The allocations across the three Small pair combinations and two Value pair combinations are averaged to determine the overall optimal allocation to the respective factor. **Exhibit 8** includes the average optimal allocations when focusing on the growth in either nominal wealth (Panel A) or real wealth (Panel B).

The results are relatively similar whether wealth is measured in nominal or real (i.e., inflation-adjusted) terms, although allocations are slightly higher to each factor when wealth is measured in real terms. Allocations to Small and Value are relatively significant (i.e., well above 50%) and increase notably as the investment period increases, roughly from 80% for Value and 60% for Small for a 1-year term up to effectively 100% for both given a 10-year investment

<sup>3</sup>See [www.morningstar.com/content/dam/marketing/apac/au/pdfs/Legal/Stylebox\\_Factsheet.pdf](http://www.morningstar.com/content/dam/marketing/apac/au/pdfs/Legal/Stylebox_Factsheet.pdf).

<sup>4</sup>See [https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data\\_library.html](https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html).

## Exhibit 8. Average Optimal Allocation to Small and Value by Investment Period and Wealth Definition Using Actual Historical Returns, 1927–2022



Sources: Kenneth French's Data Library; authors' calculations.

horizon. These results are not necessarily surprising, however, given the previously noted out-performance of the respective factors.

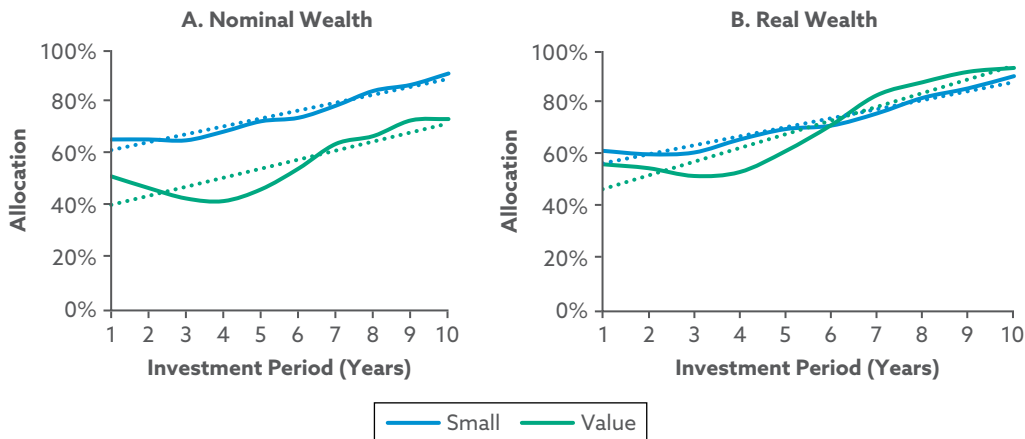
To control for the positive historical premiums, an additional set of optimizations are performed where the returns are normalized so that the average annual return and standard deviations are identical across all the series; they are set to 8% and 15%, respectively. Normalizing the historical returns effectively removes any kind of historical outperformance associated with the factors so we can better understand how the risk dynamics change by investment horizon. To clarify, since the returns and standard deviations are identical, the differences in allocations will be driven by such things as the autocorrelation levels and, for the real wealth analysis, the relationship to inflation. The optimization results using the normalized returns are presented in **Exhibit 9**.

Not surprisingly, the initial allocations to Small and Value decline when the historical out-performance is eliminated; however, there is still evidence of a benefit from allocating to the asset classes given how the allocations to Small and Value increase by investment period. For example, for an investment period of 10 years, the allocations to both Small and Value would exceed 90% if wealth is measured in real terms (which is likely a better metric than nominal wealth). While it is likely the allocations could be driven by other effects that are not explicitly controlled for when we normalize the returns (e.g., the third or fourth moments), the results demonstrate that the potential value of allocation to various asset classes (or factors) could vary by time horizon, especially when considering inflation risks.

## Allocating to Commodities

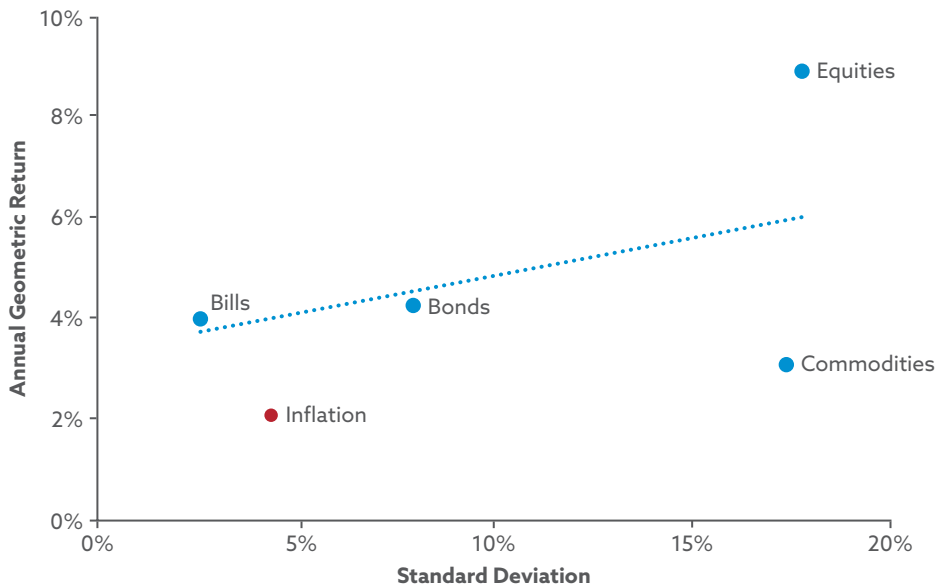
Real assets, such as commodities, often appear to be relatively inefficient within a larger opportunity set of choices and, therefore, commonly receive little (or no) allocation in common portfolio optimization routines, such as MVO. This historical inefficiency of commodities is documented quite clearly in **Exhibit 10**, which includes the historical annualized returns for US cash,

Exhibit 9. Average Optimal Allocation to Small and Value by Investment Period and Wealth Definition Using Normalized Returns, 1927–2022



Sources: Kenneth French’s Data Library; authors’ calculations

Exhibit 10. Historical Standard Deviation and Geometric Returns for Asset Classes, 1872–2023



Sources: JST Macroeconomy Database; Bank of Canada; Morningstar Direct; authors’ calculations.

US bonds, US equities, and US commodities from 1870 to 2023. The primary returns for US cash, US bonds, and US equities are obtained from the JST Macrohistory Database from 1872 (the earliest year the complete dataset is available) to 2020 (the last year available) and the Ibbotson SBBI series<sup>5</sup> thereafter.

The commodity return series uses returns from Bank of Canada (Macdonald 2017) commodity price index (BCPI) from 1872 to 1969 and the S&P GSCI Index<sup>6</sup> from 1970 to 2023. The BCPI<sup>7</sup> is a chain Fisher price index of the spot or transaction prices in US dollars of 26 commodities produced in Canada and sold in world markets. The GSCI was the first major investable commodity index and is broad-based and production-weighted to represent the global commodity market beta. The GSCI was selected due to its long history, the similarity of its component weights to those of the BCPI, and the fact that there are a number of publicly available investment products that can be used to roughly track its performance (e.g., the iShares exchange-traded fund GSG,<sup>8</sup> which has an inception date of 10 July 2006). These two commodity index proxies, in particular BCPI, are used primarily for data availability (e.g., returns going back to 1872) and familiarity, and the results from the analysis should likely be viewed with these limitations in mind.

As noted previously, commodities appear to be incredibly inefficient when compared to bills, bonds, and equities, as shown in Exhibit 10. Commodities have the same approximate annual standard deviation as equities, but they have a historical geometric return that is approximately 580 bps lower. This finding suggests that allocations to commodities would be relatively low in most optimization frameworks. Real assets, such as commodities, however, have return drivers that could be attractive to investors who are concerned about inflation. This effect was demonstrated in Exhibit 2, where the correlation between commodities and inflation increases notably for longer investment durations.

An additional series of portfolio optimizations are performed to determine how allocations to commodities would vary by investment horizon, from 1 to 10 years. Again, optimal allocations are assumed to be those that maximize the certainty equivalent of ending wealth using a utility function that assumes CRRA. We test risk aversion coefficients that effectively result in approximate equity allocations from 5% to 100% in 5% increments. Four asset classes are included in the portfolio optimizations: bills, bonds, equities, and commodities.

**Exhibit 11** includes the optimal allocations among the four asset classes by equity allocation targets assuming a one-year investment period for nominal and real wealth in Panels A and B, respectively.

The portfolio allocations in Exhibit 11 are relatively similar, although as basic bond math and the Fisher relation would suggest,<sup>9</sup> allocations to bills tend to be slightly higher and allocations to bonds lower when wealth is defined in real (versus nominal) terms. Commodities do not receive

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<sup>5</sup>Stocks, Bonds, Bills, and Inflation (SBBI) data are available at <https://rpc.cfainstitute.org/en/research-foundation/sbbi>.

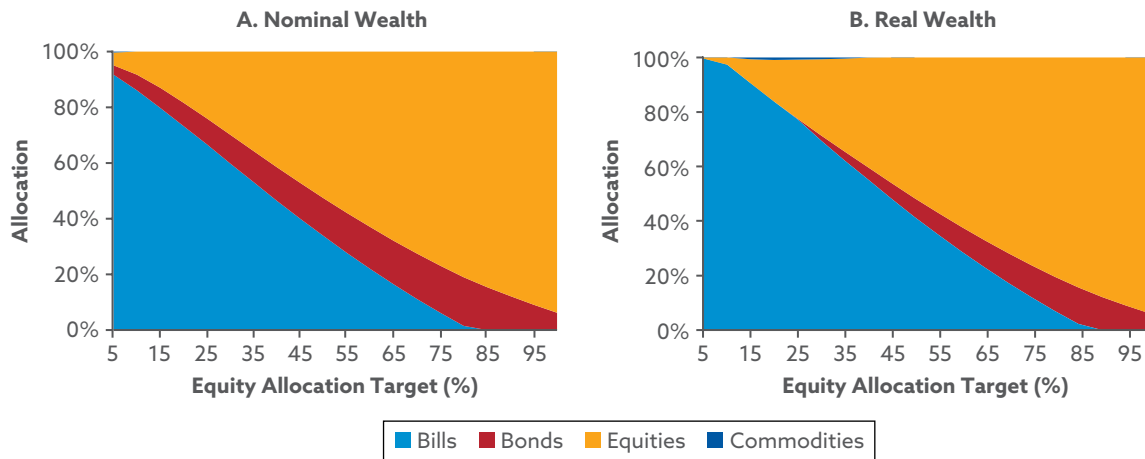
<sup>6</sup>[www.spglobal.com/spdji/en/indices/commodities/sp-gsci/](http://www.spglobal.com/spdji/en/indices/commodities/sp-gsci/).

<sup>7</sup>[www.bankofcanada.ca/rates/price-indexes/bcpi/](http://www.bankofcanada.ca/rates/price-indexes/bcpi/).

<sup>8</sup>[www.ishares.com/us/products/239757/ishares-sp-gsci-commodityindexed-trust-fund](http://www.ishares.com/us/products/239757/ishares-sp-gsci-commodityindexed-trust-fund).

<sup>9</sup>The Fisher relation is the phenomenon whereby inflation rates and interest rates tend to move together. Bond math is the fact that bond prices (but not bill prices) fall when interest rates rise.

## Exhibit 11. Optimal Asset Class Allocations for a One-Year Period, 1872–2023



Sources: JST Macrohistory Database; Bank of Canada; Morningstar Direct; authors' calculations.

any kind of meaningful allocation at all, which is consistent with expectations given the relative inefficiency noted in Exhibit 10.

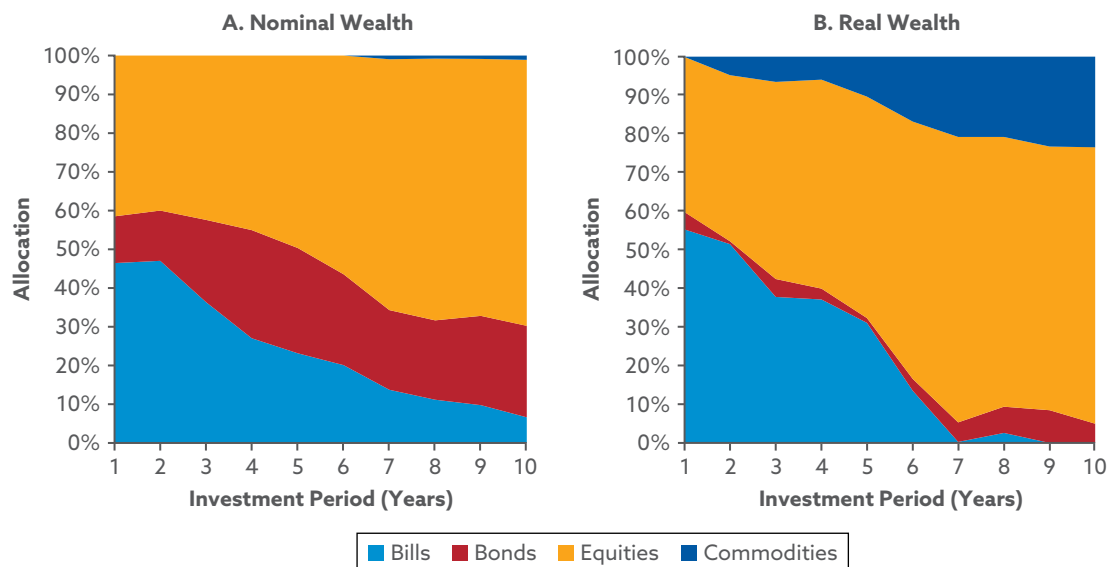
Next, in **Exhibit 12**, we provide information about how the allocations to the four asset classes vary as the time horizon increases from 1 year to 10 years in one-year increments for nominal wealth and real wealth in Panel A and Panel B, respectively, assuming a 40% equity allocation target.

We can see that while the allocations to commodities remain below 1% when wealth is defined in nominal terms (Panel A of Exhibit 12) regardless of the investment period, the allocations to commodities increase notably over time when wealth is defined in real terms (Panel B of Exhibit 12). When contrasting the two panels, transitioning from nominal wealth to real wealth results in significantly lower allocations for bonds for all holding periods, higher allocations to bills for shorter periods, and notably higher allocations to commodities as the investment term increases.

Finally, in **Exhibit 13**, we provide context about the allocations specifically to commodities for varying investment periods, equity allocation targets, and nominal and real wealth definitions in Panels A and B, respectively.

While the allocation to commodities remains at approximately zero at longer time horizons for virtually all equity allocation targets when wealth is defined in nominal returns (Panel A of Exhibit 13), when wealth is defined in real terms (Panel B of Exhibit 13), the allocations can be relatively significant over longer investment periods. This is especially true for investors targeting moderately conservative portfolios (e.g., ≈40% equity allocations), where optimal allocations to commodities would be roughly 20%. In other words, the perceived historical benefits of allocating to commodities has varied significantly depending on the definition of wealth (nominal versus real) and the assumed investment period (e.g., moving from 1 year to 10 years).

## Exhibit 12. Optimal Asset Class Allocations by Investment Period Assuming a 40% Equity Risk Target, 1872–2023



Sources: JST Macroeconomy Database; Bank of Canada; Morningstar Direct; authors' calculations.

Additionally, while commodities have significantly underperformed equities in the past (by approximately 600 bps), according to the Horizon 2023 Capital Market Assumptions survey, future expected 10-year returns for commodities are approximately only 200 bps lower than those for equities, at 4.9% and 6.9%, respectively (Horizon Actuarial Services 2023). These findings suggest that on a forward-looking basis, commodities could be even more attractive than when using purely historical returns.

## Conclusion

While describing the risk of an asset using expected returns and covariance is obviously a simplifying assumption (albeit one that MVO makes and many people use without recognizing its limitations), this research demonstrates that ignoring serial dependencies in asset class returns is likely to result in portfolio allocations that are notably different than if they were considered. Forecasting the various dependencies that have existed (or could exist) is relatively complicated, especially across asset classes and given the base estimation error implicit with any kind of forecasting. Therefore, although issues surrounding serial dependence may need to be more qualitatively than quantitatively incorporated, they still need to be considered to ensure portfolios are as efficient as possible, especially for longer-term investors and investors concerned with inflation.

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### Exhibit 13. Optimal Allocation to Commodities by Wealth Definition, Equity Risk Target, and Investment Period, 1872–2023

A. Nominal Wealth											B. Real Wealth											
Investment Period (Years)											Investment Period (Years)											
	1	2	3	4	5	6	7	8	9	10		1	2	3	4	5	6	7	8	9	10	
Equity Risk Target	5	0	1	0	4	4	4	3	0	0		5	0	4	2	0	0	0	10	29	35	12
Equity Risk Target	15	0	0	0	0	0	0	0	0	0		15	1	3	2	0	0	4	12	21	24	15
Equity Risk Target	25	0	0	0	0	0	0	0	0	0		25	1	5	7	0	4	14	18	19	22	21
Equity Risk Target	35	0	0	0	0	0	1	0	0	1		35	0	5	7	5	9	17	21	20	24	24
Equity Risk Target	45	0	0	0	0	0	1	1	1	1		45	0	4	6	6	11	17	20	20	22	22
Equity Risk Target	55	0	0	0	0	0	1	1	1	1		55	0	4	5	6	10	15	17	17	19	19
Equity Risk Target	65	0	0	0	0	0	0	0	0	0		65	0	3	4	5	9	13	13	13	14	14
Equity Risk Target	75	0	0	0	0	0	0	0	0	0		75	0	2	3	3	7	9	9	9	10	11
Equity Risk Target	85	0	0	0	0	0	0	0	0	0		85	0	1	1	1	4	5	5	6	7	7
Equity Risk Target	95	0	0	0	0	0	0	0	0	0		95	0	0	0	0	1	1	2	2	3	3

Sources: JST Macrohistory Database; Bank of Canada; author's calculations.

# APPENDIX 1. OLS REGRESSION RESULTS

For this analysis, we run a series of ordinary least-squares regressions using historical bill rates, bond returns, and equity returns for 16 countries using data from the JST Macrohistory Database, where returns are available only up to 2020. The specific countries and number of historical years available for the tests are included in **Exhibit A1.1**.

**Exhibit A1.2** includes a summary of the results. As opposed to reporting the actual coefficients, we report the sign of the coefficient (positive or negative) along with an indication of statistical significance. For example, +++ would be a positive coefficient significant at the 0.01% level, ++ positive at the 1% level, and + positive at the 5% level.

## Exhibit A1.1. Number of Years Included in Regressions

Abbrev.	Name	Number of Years		
		Bill	Bond	Equity
AUS	Australia	73	121	151
BEL	Belgium	102	101	151
CHE	Switzerland	151	105	121
DEU	Germany	71	72	151
DNK	Denmark	146	105	148
ESP	Spain	n/a	80	121
FIN	Finland	151	151	125
FRA	France	99	151	151
GBR	United Kingdom	151	151	150
ITA	Italy	99	151	151
JPN	Japan	145	140	73
NLD	Netherlands	151	151	121
NOR	Norway	151	151	140
PRT	Portugal	141	150	150
SWE	Sweden	151	150	150
USA	United States	151	150	149



.....

## Exhibit A1.2. Individual Country Results

	Bill Rates						Bond Returns						Equity Returns				
	t-1	t-2	t-3	t-4	t-5		t-1	t-2	t-3	t-4	t-5		t-1	t-2	t-3	t-4	t-5
AUS	+++	--				AUS				+		AUS		-			
BEL	+++					BEL						BEL	++	-			
CHE	+++					CHE						CHE					-
DEU	+++	---	+++	-		DEU						DEU					
DNK	+++	+				DNK					+++	DNK					
						ESP		+			+	ESP	++				
FIN	+++					FIN						FIN					
FRA	+++	--	+			FRA		++				FRA					
GBR	+++	---	+			GBR					++	GBR					
ITA	+++					ITA						ITA					
JPN	+++				+	JPN						JPN	+				-
NLD	+++				+	NLD						NLD					
NOR	+++	---	+			NOR				++	+	NOR					
PRT	+++	---				PRT	+		-		+	PRT	++		--		
SWE	+++					SWE	-			++	+	SWE					
USA	+++	---	+++	-	++	USA			++	+		USA		--			-

We can see that there is notable positive autocorrelation for bill rates, which is expected. Relatively few coefficients are statistically significant when focusing on bond returns and equity returns; however, the bond return coefficients that are statistically significant are generally positive and the equity return coefficients that are statistically significant are generally negative.

# APPENDIX 2. EXPANDING THE BOOTSTRAP ANALYSIS TO INTERNATIONAL MARKETS

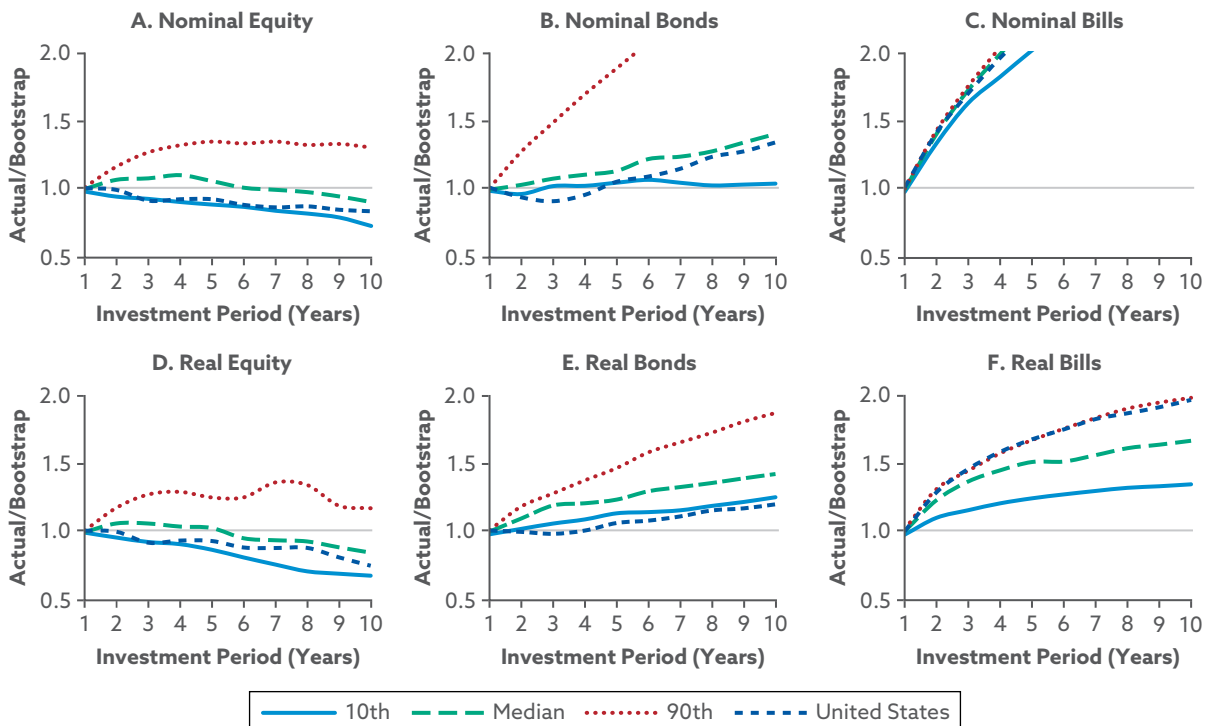
This section extends the analysis conducted for Exhibit 3 by focusing on international returns. The analysis focuses on the ratio of the actual standard deviation of wealth for the historical time series versus what would be expected using bootstrapped returns.

The analysis is extended to 14 countries other than the United States, using only returns from the JST Macrohistory Database. We do not include Canada or Ireland, because data are not available, and we do not include Germany given the notable returns in the 1920s and 1940s. **Exhibit A2.1** includes the 10th percentile, median, and 90th percentile ratios for equities, bonds, and bills, both in nominal and real returns, for the countries considered; US data are included for reference purposes.

The results in Exhibit A2.1 are relatively consistent with those shown in Exhibit 3, whereby the risk of equities generally declines over longer periods (versus bootstrapped returns) while the risk of bonds and bills generally rises. While the effects vary by country, the results do imply there have been important time-varying effects for international markets as well.



## Exhibit A2.1. Ratio of Actual to Bootstrapped Standard Deviation of Wealth, 1850–2020



## APPENDIX 3. PAIRS TESTED FOR THE RESPECTIVE FACTORS

Factor	Asset Class 1	Asset Class 2
Small	Small Growth	Large Growth
Small	Small Blend	Large Blend
Small	Small Value	Large Value
Value	Small Value	Small Growth
Value	Large Value	Large Growth

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